

# Uncertainties on Central Exclusive Scalar Luminosities from the unintegrated gluon distributions.

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**ABSTRACT:** In a previous report we used the Linked Dipole Chain model unintegrated gluon densities to investigate the uncertainties in the predictions for central exclusive production of scalars at hadron colliders. Here we expand this investigation by also looking at other parameterizations of the unintegrated gluon density, and look in more detail on the behavior of these at small  $k_{\perp}$ . We confirm our conclusions that the luminosity function for central exclusive production is very sensitive to this behavior. However, we also conclude that the available densities based on the CCFM and LDC evolutions are not constrained enough to give reliable predictions even for inclusive Higgs production at the LHC.

**KEYWORDS:** QCD, Jets, Parton Model, Phenomenological Models.

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## 1. Introduction

Detecting the Higgs boson at LHC in the “most probable” mass region around 120 GeV is far from a trivial task, such a light Higgs predominantly decays into bottom quarks making the background from standard QCD processes huge. Looking for Higgs signals in the clean environment of central diffractive events is therefore an appealing prospect, provided the cross section is sufficiently high [1–8].

In general, central exclusive events can be used for studying any scalar particle. In this paper we will only consider a Higgs boson, but our results can be trivially generalized. Central exclusive diffractive Higgs production was first suggested in [1,2] and has lately been developed further by Khoze, Martin and Ryskin (KhMR)<sup>1</sup> [5]. One of the main advantages compared to inclusive Higgs production is that, since the central system is constrained to be in a  $0^{++}$  state, the normal QCD background from b-jets is heavily suppressed. By matching the mass of the central system, as measured with the central detectors, with the mass calculated from the energy loss of the scattered protons detected by very forward proton taggers, it is possible to exclude events with extra radiation outside the reach of the detectors, to ensure that the central system is indeed in a  $0^{++}$  state.

In [10] we investigated the implications of the uncertainties in the unintegrated structure functions, uPDFs, for the KhMR calculations. Our main conclusion was that the cross section is very sensitive to the unintegrated structure functions,  $G(x, k_{\perp}^2, m_H^2)$ , in the

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<sup>1</sup>We shall here refer to their calculations as KhMR to distinguish it from the KMR procedure for obtaining unintegrated gluon densities from integrated ones by Kimber, Martin and Ryskin [9].

region of  $k_{\perp} \approx 2 - 3$  GeV. The differences in the uPDF, which enters in the final exclusive luminosity to the power of four, leads to a variation in the result of roughly one order of magnitude. This estimate was obtained using unintegrated structure functions both from KMR [9] (used in the KhMR calculations) and different parameterizations based on the Linked Dipole Chain model, LDC [11].

In this report we have also used the CCFM-based densities described in [12], here referred to as Jung-1 and Jung-2. Both LDC and Jung have been tuned to  $F_2$  data from HERA in the region of small  $x \lesssim 0.01$  and  $1.5 < Q^2 \lesssim 100$  GeV<sup>2</sup>. Despite the similar fitting region and the similarities between CCFM and LDC evolution, it is found that the densities differ substantially in their  $k_{\perp}$ -distribution [11] even inside the fitting region. This is to be expected, since the fitting was only done to  $F_2$ , which is an integrated quantity. Below we will find that the differences at high scales, corresponding to the production of a 120 GeV Higgs, is large even for the integrated density. This can be explained by the fact that here the densities are also influenced by the large  $x$  distribution at smaller scales, well outside the region of the fit.

In the KMR case the uPDFs are derived directly from the globally fitted integrated gluon density, MRST98 [13]<sup>2</sup>. Hence at least the integrals of the uPDFs are well constrained. On the other hand, the  $k_{\perp}$  dependence is uncertain since KMR assumes DGLAP evolution which works well for inclusive observables but not necessarily for  $k_{\perp}$  sensitive ones.

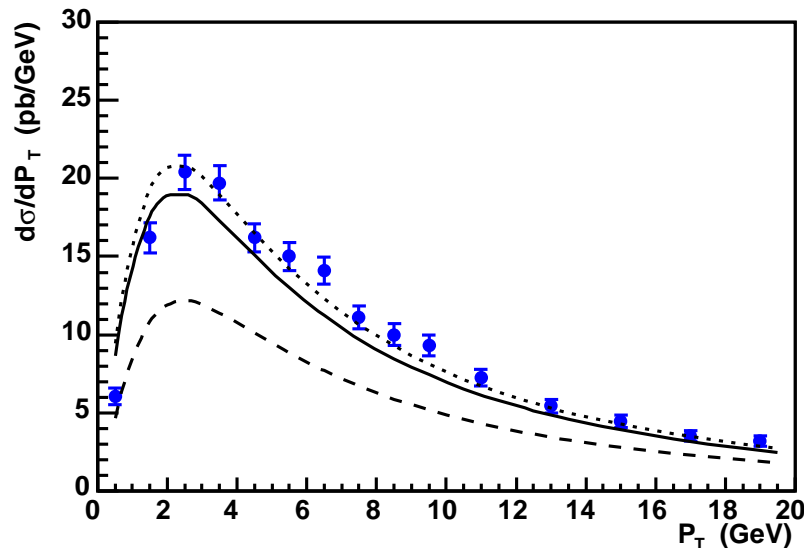
The off-diagonal unintegrated parton densities (oduPDFs) which enters into the KhMR calculations were derived in [14] from the corresponding off-diagonal integrated one (odPDF) in the same way as the uPDFs were derived from the standard integrated PDFs in [13]. Now, while the integrated gluon PDF is fairly well constrained experimentally, the unintegrated is not, and the off-diagonal unintegrated, used in the exclusive cross section, is even less so. And any uncertainty in the uPDF will immediately be reflected in an uncertainty in the oduPDF.

There are a few weak experimental constraints on the  $k_{\perp}$ -distribution of the uPDFs. So far these constraints have not been taken into account in any fitting, but comparing models using the uPDFs with data can give us some hints about where the densities work and where they need to be improved. Since the exclusive luminosity is sensitive to the uPDF mainly in the region of a few GeV we should look for other observables sensitive to features in this region to obtain constraints. One such observable is the  $k_{\perp}$ -spectra of W and Z in hadron collisions (eg. at the Tevatron [15, 16]) for small  $k_{\perp}$ . While the main features of this can be reproduced by a calculation using KMR uPDFs [17], the small- $k_{\perp}$  peak is slightly too low, as can be seen in figure 1, indicating that KMR may be underestimating somewhat the hardness of the  $k_{\perp}$ -distribution.

Another sensitive observable is the rate of forward jets in DIS at HERA. Especially in the measured region of  $k_{\perp}^2 \sim Q^2 \gtrsim 10$  GeV<sup>2</sup> and small  $x$  where standard DGLAP evolution would not contribute. Indeed, DGLAP based models severely underestimate the rate of

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<sup>2</sup>MRST98 is not the newest of PDF parameterizations, but it was used in [5], where it was also shown that the results are rather insensitive to the choice of integrated PDF.



**Figure 1:** The  $p_{\perp}$ -distribution of  $Z^0$  measured at the Tevatron [15] compared to a calculation using the KMR approach to uPDFs with different options as described in [17].

forward jets (see eg. [18] and [19] for a discussion on this), and even though the KMR uPDFs have not been confronted with this data it is likely that they will also fail.

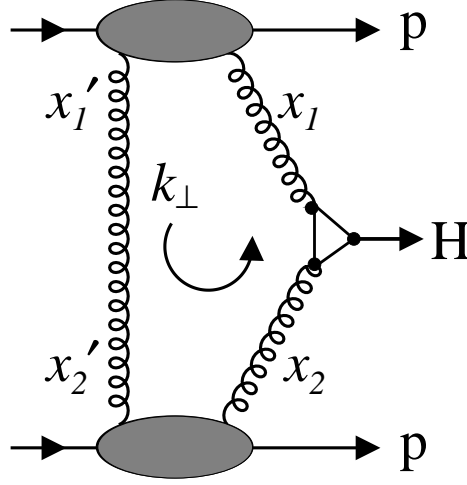
In general there are indications of a slightly harder  $k_{\perp}$  distribution in the uPDFs than what is given by KMR. This is predicted by the BFKL-like CCFM evolution (and hence also LDC) on which the alternative uPDFs used in this report are based on. Such evolution includes also ladders unordered in transverse momenta, opening up for more activity. As shown in [20] the typical evolution path, starting from the high virtuality end, is a rapid DGLAP-like evolution down to a few GeV and then a region of transverse momenta distributed as a random walk in  $\log(k_{\perp})$ .

The layout of this paper is as follows. First we recapitulate the main points in the calculation of Khoze, Martin and Ryskin and discuss their oduPDFs in section 2. In section 3 we obtain the oduPDFs in the case of LDC and Jung respectively. Then, in section 4 we present and comment our results. Finally we arrive at our conclusions in section 5.

## 2. Central exclusive production

In a central exclusive production of a Higgs boson, two gluons with no net quantum number fuse into a Higgs via the standard heavy quark triangle diagram, whereas another semi-hard gluon exchange guarantees that there is no net colour flow between the protons. This is shown in figure 2, where it is also indicated that the exchanged semi-hard gluon should compensate the transverse momentum  $k_{\perp}$  of the gluons producing the Higgs, so that the protons are scattered with little or no transverse momenta.

Several types of radiation can destroy the diffractive character of the interaction. There can be extra interactions between the spectator partons, modeled by the so called soft survival probability  $S^2$ . Also, the gluons participating in the interaction can radiate both at scales above  $k_{\perp}$ , which is modeled by the hard survival probability using a Sudakov form



**Figure 2:** The basic diagram for exclusive production of the Higgs boson in hadron collisions.

factor, and at scales below  $k_\perp$ , which is suppressed, since such gluons cannot resolve the individual colours of the exchanged gluon pair.

This is discussed in detail in [21] and [10]. Here we just state the resulting exclusive luminosity function

$$\begin{aligned}
 L(M, y) &= \frac{\delta^2 \mathcal{L}}{\delta y \delta \ln M^2} \\
 &= S^2 \left[ \frac{\pi}{(N_c^2 - 1)b} \int^{\mu^2} \frac{dk_\perp^2}{k_\perp^4} f_g(x_1, x'_1, k_\perp^2, \mu^2) f_g(x_2, x'_2, k_\perp^2, \mu^2) \right]^2
 \end{aligned} \tag{2.1}$$

where  $\mu^2 = M^2/4$  in the standard KhMR prescription,  $y$  denotes rapidity,  $b$  comes from the probability for the protons to remain intact,  $x_{1(2)} = m_H e^{(-)y}$  and  $x'_{1(2)} \sim k_\perp / \sqrt{S} \ll x_{1(2)}$ .  $f(x, x', k_\perp^2, \mu^2)$  is the off-diagonal unintegrated gluon density, the oduPDF, which should be interpreted as the amplitude related to the probability of finding two gluons in a proton with equal but opposite transverse momentum,  $k_\perp$ , and carrying energy fractions  $x$  and  $x'$  each, one of which is being probed by a hard scale  $\mu^2$ .

The cross section is then obtained by

$$\sigma = \int \hat{\sigma}_{gg \rightarrow H}(M^2) \frac{\delta^2 \mathcal{L}}{\delta y \delta \ln M^2} dy d \ln M^2$$

where  $M$  is the invariant mass of the central system, in this case the Higgs mass. In principle one should use a off-shell version of  $\hat{\sigma}$  (see eg. [22]) which then would have a  $k_\perp$  dependence, and hence break the factorization. However, for the exclusive cross section the main contribution comes from rather small  $k_\perp$  and, at least for large masses, the factorization should hold. Since the cross section, in the exclusive case, is a convolution of

the luminosity and the matrix element it suffices to study the difference in luminosity to investigate the effects of different oduPDFs.

Besides the oduPDFs, the only other main uncertainty in eq. (2.1) is the soft survival probability  $S^2$ . We have made a separate study using the multiple interaction model in PYTHIA [23, 24] in the same way as was done for the  $WW \rightarrow H$  process in [25]. Taking the probability of having no additional scatterings in Higgs production<sup>3</sup> using the parameters of the so-called Tune-A by Rick Field [26], we estimate the survival probability to be 0.040 for the Tevatron and 0.026 for the LHC. This is remarkably close to the values used in [5] obtained in the so-called two-channel eikonal approach [27].

In the KhMR case the oduPDF [5] are obtained in a two step procedure presented in [14]. In the first step the off-diagonal parton distribution functions, odPDF, are extracted from the standard gluon PDF, in the relevant limit of  $x' \ll x$ :

$$H(x, x', \mu^2) \approx R_g x g(x, \mu^2). \quad (2.2)$$

Although we will use a constant  $R_g$  factor of 1.2, we note that it in general depends on both  $x$  and  $\mu^2$ . The consequences for the luminosity function of a non-constant  $R_g$  are moderate and briefly discussed in [10].

In the second step it is assumed that the oduPDF can be obtained from the odPDF in the same way as the uPDF can be obtained from the standard PDF. In the latter case one can use the KMR prescription introduced in [9], where

$$G(x, k_\perp^2, \mu^2) \approx \frac{d}{d \ln k_\perp^2} [x g(x, k_\perp^2) T(k_\perp^2, \mu^2)], \quad (2.3)$$

which then corresponds to the probability of finding a gluon in the proton with transverse momentum  $k_\perp$  and energy fraction  $x$  when probed with a hard scale  $\mu^2$ .  $T$  is the survival probability of the gluon given by the Sudakov form factor,

$$\ln T(k_\perp^2, \mu^2) = - \int_{k_\perp^2}^{\mu^2} \frac{dq_\perp^2}{q_\perp^2} \frac{\alpha_S(q_\perp^2)}{2\pi} \int_0^{\frac{\mu}{\mu+q_\perp}} dz [z P_g(z) + n_f P_q(z)]. \quad (2.4)$$

To get the oduPDF one then starts from eq. (2.2) and get by analogy in the limit  $x' \ll x$

$$f_g(x, x', k_\perp^2, \mu^2) \approx \frac{d}{d \ln k_\perp^2} \left[ R_g x g(x, k_\perp^2) \sqrt{T(k_\perp^2, \mu^2)} \right], \quad (2.5)$$

where the square root of the Sudakov comes about because only one of the two gluons are probed by the hard scale.

The hard scale  $\mu$  in the oduPDF and in the Sudakov form factor is in the KhMR approach argued to be  $m_H/2$ . In fact the number is  $0.62 \cdot m_H$  and comes from a tuning to reproduce full one-loop vertex corrections [28]. For LDC, below, we will be less ambitious and simply use  $m_H$  as scale.

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<sup>3</sup>We here used inclusive Higgs production, but the result should be the same for the exclusive case

### 3. Unintegrated parton densities

A general comment concerning the unintegrated gluon densities used in the KhMR calculations is that the KMR prescription essentially corresponds to taking one step backward in a DGLAP-based initial-state parton shower, to *unintegrate* the integrated PDF. As mentioned in the introduction, there are indications that such a prescription underestimates the hardness of the  $k_\perp$ -distribution of the uPDF. Therefore we will here investigate uPDFs based on CCFM and LDC evolution, where emissions unordered in  $k_\perp$  are allowed, which could increase the hardness of the  $k_\perp$ -distribution.

#### 3.1 The Linked Dipole Chain uPDF

The Linked Dipole Chain model [29,30] is a reformulation and generalization of the CCFM [31–34] evolution for the unintegrated gluon. CCFM has the property that it reproduces BFKL evolution [35,36] for asymptotically large energies (small  $x$ ) and is also similar to standard DGLAP evolution [37–40] for larger virtualities and larger  $x$ . It does this by carefully considering coherence effects between gluons emitted from the evolution process, allowing only gluons ordered in angle to be emitted in the initial state, and thus contribute to the uPDFs, while non-ordered gluons are treated as final state radiation off the initial state gluons. LDC differs from CCFM by the fact that it is ordered both in positive and negative light cone momenta,  $q_+$  and  $q_-$ , of the emitted gluons, a treatment which categorizes more emissions as final state emission as compared to CCFM. This symmetric ordering in both  $q_+$  and  $q_-$ , which also implies ordering in rapidity  $y$  or angle, together with the additional requirement that the transverse momentum of an emitted gluon must be larger than the  $k_\perp$  of the propagator gluon before or after the emission, greatly simplifies the evolution equations and has as a consequence that the uPDF approximately factorizes into a one-scale density multiplied by the Sudakov form factor:

$$G(x, k_\perp^2, \mu^2) \approx G(x, k_\perp^2) \times \Delta_S(k_\perp^2, \mu^2), \quad (3.1)$$

where

$$\ln \Delta_S(k_\perp^2, M^2) = - \int_{k_\perp^2}^{M^2} \frac{dq_\perp^2}{q_\perp^2} \frac{\alpha_s}{2\pi} \int_0^{1-q_\perp/M} dz \left[ z P_g(z) + \sum_q P_q(z) \right]. \quad (3.2)$$

The LDC model has been implemented in an event generator which is then able to generate complete events in DIS with final state radiation added according to the dipole cascade model [41,42] and hadronization according to the Lund model [43]. One advantage of having an event generator implementation is that energy and momentum can be conserved in each emission. Since the lack of momentum conservation in the BFKL formalism is the main reason for the huge next-to-leading logarithmic corrections [44], the LDC model is therefore expected to have smaller sub-leading corrections (see [18] for a more detailed discussion on this).

The perturbative form of the uPDF needs to be convoluted with non-perturbative input PDFs, the form of which are fitted to reproduce the experimental data on  $F_2$ . This has all been implemented in the LDCMC program [45,46], and the resulting events can be compared

directly to experimental data from eg. HERA. The LDC gluon uPDF can then be extracted by generating DIS events with LDCMC and measuring the gluon density as described in [11]. Due to the  $k_\perp$ -unordered nature of the LDC evolution, the relationship between the uPDF and the standard gluon density is different from eq. (2.3), as the integrated gluon at a scale  $\mu^2$  also receives a contribution, although suppressed, from gluons with  $k_\perp > \mu$ , and in [11] the following expression was obtained:

$$xg(x, \mu^2) = G_0(x)\Delta_S(k_{\perp 0}^2, \mu^2) + \int_{k_{\perp 0}^2}^{\mu^2} \frac{dk_\perp^2}{k_\perp^2} G(x, k_\perp^2) \Delta_S(k_\perp^2, \mu^2) + \int_{\mu^2}^{\mu^2/x} \frac{dk_\perp^2}{k_\perp^2} G(x \frac{k_\perp^2}{\mu^2}, k_\perp^2) \frac{\mu^2}{k_\perp^2}, \quad (3.3)$$

where  $G_0(x)$  is the non-perturbative input parameterization at the cutoff scale  $k_{\perp 0}$ .

Note that a sharp cutoff  $k_{\perp 0}$  is assumed, which could cause problems in calculations sensitive to the small- $k_\perp$  behavior. To avoid this we redefine the uPDF as

$$G(x, k_\perp^2, \mu^2) = \begin{cases} a \left( \frac{k_\perp^2}{k_{\perp 0}^2} \right)^a G_0(x) \Delta_S(k_{\perp 0}^2, \mu^2) & k_\perp < k_{\perp 0} \\ G(x, k_\perp^2) \Delta_S(k_\perp^2, \mu^2) & k_{\perp 0} < k_\perp < \mu \\ G(x \frac{k_\perp^2}{\mu^2}, k_\perp^2) \frac{\mu^2}{k_\perp^2} & \mu < k_\perp < \mu/\sqrt{x} \end{cases}, \quad (3.4)$$

where  $a$  can either be set to 1, as was effectively done in [10], or to  $G(x, k_{\perp 0}^2)/G_0(x)$  which makes the distribution continuous across  $k_{\perp 0}$ . In this way we get the standard form

$$xg(x, \mu^2) = \int_0^\infty \frac{dk_\perp^2}{k_\perp^2} G(x, k_\perp^2, \mu^2), \quad (3.5)$$

and we find that our results are not very sensitive to the choice of  $a$ .

To obtain the off-diagonal densities needed for the exclusive luminosity function, we assume that a similar approximation can be made as for the KMR densities, that is, in the limit of very small  $x'$

$$f_g^{\text{LDC}}(x, x', k_\perp^2, \mu^2) \approx R_g G(x, k_\perp^2) \sqrt{\Delta_S(k_\perp^2, \mu^2)}. \quad (3.6)$$

The square root of the Sudakov form factor is used, since only one of the gluons couples to the produced Higgs at the high scale, and we could equivalently have written

$$f_g^{\text{LDC}}(x, x', k_\perp^2, \mu^2) \approx R_g \sqrt{G(x, k_\perp^2, \mu^2) G(x, k_\perp^2, k_\perp^2)}. \quad (3.7)$$

We note that this is not completely equivalent to eq. (2.5), but it is a prescription which can be used for any uPDF, not only the KMR one. Using eq. (3.7) rather than eq. (2.5) for the KMR uPDFs we find that the exclusive luminosity function is underestimated by  $\approx 50\%$  for a higgs mass of 120 GeV.



### 3.2 The Jung 2003 uPDF parameterizations

The Jung 2003 [12] unintegrated parton distribution functions are based on standard CCFM evolution and was obtained using a Monte Carlo implementing forward evolution<sup>4</sup>. The main difference w.r.t. LDC is, as mentioned above, that CCFM allows more emissions in the initial state, which makes it more infrared sensitive and which prevents the simple factorization into a one-scale density and a Sudakov form factor as in eq. (3.1). Another difference is that CCFM only describes gluon evolution, while in the LDC it is also possible to include quarks.

Just as for LDC, the perturbative CCFM evolution needs to be convoluted with non-perturbative input parton density, the parameters of which are determined by a fit to  $F_2$  at small  $x$  determined at HERA.

To produce full events the Jung uPDFs may be convoluted with an appropriate off-shell matrix element (eg.  $\gamma^* g^* \rightarrow q\bar{q}$ ) and the final state partons can then be generated in a backward evolution algorithm implemented in the CASCADE program [49].

To obtain the off-diagonal densities, we use the same procedure as in LDC given by eq. (3.7),

$$f_g^{\text{Jung}}(x, x', k_\perp^2, \mu^2) \approx R_g \sqrt{G(x, k_\perp^2, \mu^2) G(x, k_\perp^2, k_\perp^2)}, \quad (3.8)$$

but note that the equivalence with eq. (3.6) does not hold since the factorization in eq. (3.1) is absent in the Jung uPDFs.

### 3.3 Summary of uPDFs

Within the three different procedures for obtaining uPDFs there are a number of optional behaviors to choose from which are summarized in table 1. For KMR we can choose different integrated densities to start from, but that has already been shown to only give rise to moderate differences [5]. Since the integrated PDFs have been fitted to a wide range of inclusive data, the description of such observables are trivially also reproduced by the KMR uPDFs. For less inclusive observables the situation is less clear, and as argued in the introduction there are indications that the KMR procedure will underestimate slightly the hardness of the  $k_\perp$ -distribution especially at small  $k_\perp$ . And although it has been showed to be able to reproduce inclusive jet cross sections in deeply inelastic scattering at HERA [50], it is not likely that it will be able to explain the forward jet rates with  $k_{\perp \text{jet}}^2 \sim Q^2$ .

In [10] we used the three different options for the LDC densities introduced in [11], which differ in the splitting functions included in the evolution. The *standard* option includes all splitting functions and hence includes also the evolution of quarks. The *gluonic* and *leading* options only includes gluons and differs in that the latter only includes the leading  $1/z$  and  $1/(1-z)$  terms in the gluon splitting function. All give reasonable fits to HERA  $F_2$  measurements in the region  $x < 0.01$  and  $1 \text{ GeV}^2 \lesssim Q^2 \lesssim 100 \text{ GeV}^2$ . The *standard* option also describes  $F_2$  at higher  $x$  values where the contribution of valence quarks is more important. Of the three only the *leading* is able to satisfactorily describe

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<sup>4</sup>Based on the SMALLX program [47, 48].

uPDF	evolution	splittings	inclusive observables	forward jets
KMR	DGLAP	full	globally good fit	probably not
<i>standard</i>	LDC	full	HERA $F_2$	no
<i>gluonic</i>	LDC	full gluon	HERA $F_2$ at small $x$	no
<i>leading</i>	LDC	singular gluon	HERA $F_2$ at small $x$	yes
Jung-1	CCFM	singular gluon	HERA $F_2$ at small $x$	yes
Jung-2	CCFM	full gluon	HERA $F_2$ at small $x$	no

**Table 1:** Summary of the different uPDFs used in this report, indicating the differences in evolution and the ability to reproduce experimental observables.

forward jets indicating that the other two probably underestimates the hardness of the  $k_\perp$ -distribution of the gluon.

The Jung 2003 distributions also come with different options. Here we will use Jung-1 and Jung-2 which are similar to the LDC *leading* and *gluonic* options respectively in that the former only uses the leading terms in the gluon splitting functions, while the latter uses the full splitting function. Also these give a good description of  $F_2$  in the fitted region of  $x < 0.01$  and  $1 \text{ GeV}^2 \lesssim Q^2 \lesssim 100 \text{ GeV}^2$ . When used in the CASCADE generator, only the Jung-1 is able to give a good description of forward jets. Also for other observables the Jung uPDFs give results which are consistent with the ones obtained with LDC.

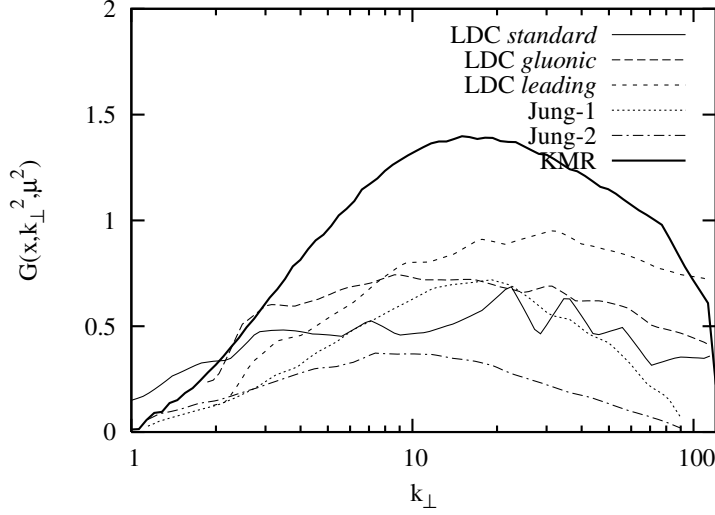
## 4. Results

Armed with these six uPDFs and their corresponding off-diagonal densities, we now want to see how they influence the exclusive luminosity function at LHC energies. But before we do this we want to compare the uPDFs in general to see if they at all make sense at the scales involved when considering Higgs production at LHC.

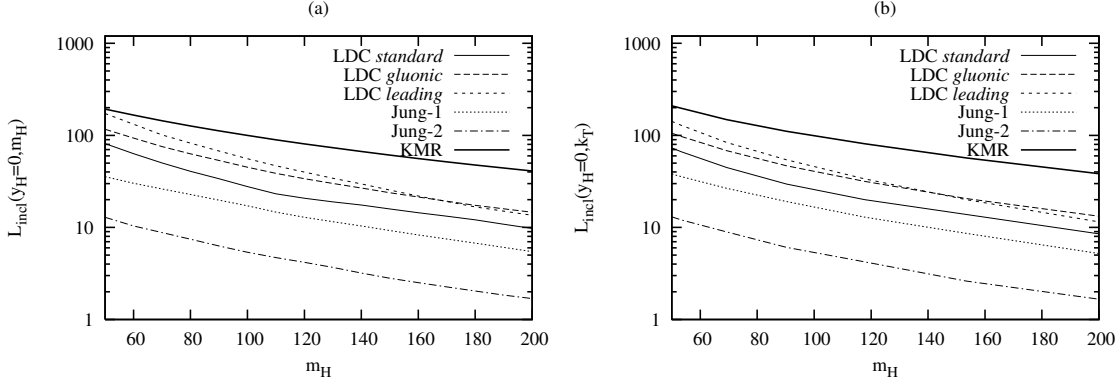
### 4.1 Inclusive Higgs production

First we look in figure 3 at the uPDFs relevant for producing a central exclusive 120 GeV Higgs at the LHC, i.e.  $x = x_H = m_H/\sqrt{S}$  and  $\mu = m_H = 120 \text{ GeV}$ . What is shown is the logarithmic density in  $k_\perp$  and clearly there are large differences between the uPDFs both in shape and normalization. For the shape the LDC densities stick out as they do not tend to zero for  $k_\perp \rightarrow \mu$ . This is as expected for LDC evolution with unordered  $k_\perp$ -evolution. CCFM will also allow  $k_\perp > \mu$ , but it seems that this is more suppressed for high scales. For the shapes we can also imagine a rough agreement between *standard*, *gluonic* and Jung-2, while *leading* and Jung-1 are clearly harder. This is also expected as the absence of nonsingular terms in *leading* and Jung-1 enhances the radiation from gluons.

The difference in normalization also shows up in the predictions for inclusive Higgs production. This is shown in figure 4. In figure 4a we show the square of the integrated gluon densities, which would enter in a calculation using collinear factorization. In figure



**Figure 3:** The unintegrated gluon densities as a function of  $k_\perp$  for  $\mu = m_H = 120$  GeV and  $x = x_H = m_H/\sqrt{S} \approx 0.086$ . The full line is LDC *standard*, long-dashed is LDC *gluonic*, short-dashed LDC *leading*, dotted is Jung-1, dash-dotted is Jung-2 and the thick full line is KMR. The wiggly shape of the LDC curves is due to low statistics when extracting them in [11].



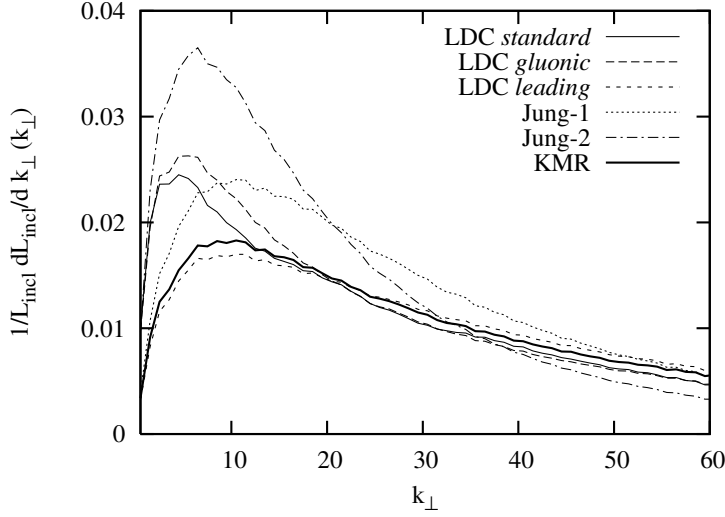
**Figure 4:** The inclusive gluon luminosity for central Higgs production as a function of  $m_H$ . (a) is simply the square of the integrated gluon density, while (b) is properly integrated over  $k_\perp$  and includes the  $k_\perp$ -dependence of the off-shell matrix element. The lines are the same as in figure 3

4b we use the  $k_\perp$ -dependence of the off-shell matrix element given in [22]:

$$\hat{\sigma}^*(m_H, \vec{k}_{\perp 1}, \vec{k}_{\perp 2}) = \hat{\sigma}_0 \cdot 2 \left( \frac{m_{\perp H}^2 \cos(\phi)}{m_H^2 + k_{\perp 1}^2 + k_{\perp 2}^2} \right)^2, \quad (4.1)$$

where  $\vec{k}_{\perp 1}$  and  $\vec{k}_{\perp 2}$  are the incoming transverse momenta,  $\phi$  the angle between them,  $m_{\perp H}$  the resulting transverse mass of the Higgs, and  $\hat{\sigma}_0$  is the standard on-shell matrix element. This gives us the inclusive luminosity function,

$$L(m_H, y) = \int \frac{dk_{\perp 1}^2}{k_{\perp 1}^2} \frac{dk_{\perp 2}^2}{k_{\perp 2}^2} d\phi \frac{\hat{\sigma}^*}{\hat{\sigma}_0} G(x_1, k_{\perp 1}^2, \mu^2) G(x_2, k_{\perp 2}^2, \mu^2), \quad (4.2)$$



**Figure 5:** The normalized  $k_{\perp}$ -distribution of a central Higgs produced at LHC as predicted by using different uPDFs. The lines are the same as in figure 3.

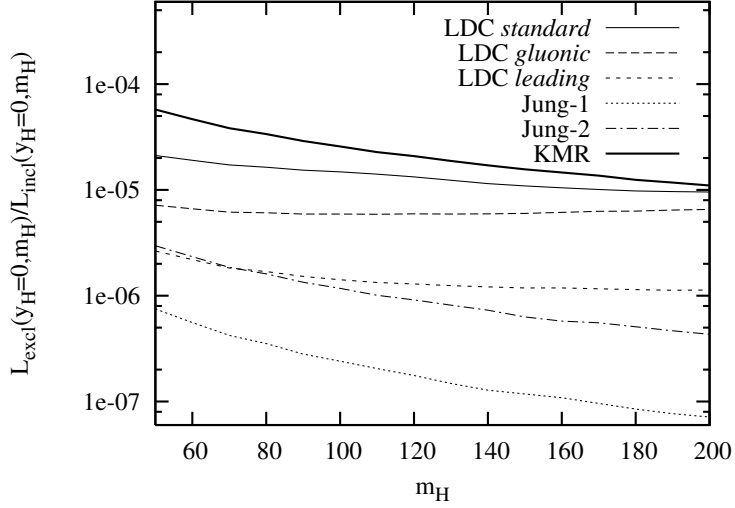
where  $x_{1,2} = m_{\perp H} e^{\pm y}$  and  $\mu = m_{\perp H}$ . We use this scale also for KMR, since this is what was used in the case of W and Z production [17]. As seen in figure 4 there are small, but not insignificant differences between the collinear and off-shell versions. In fact the off-shell version takes into account some of the beyond leading order effects which are absent in our LO collinear approximation.

Clearly the differences in the inclusive luminosity are too large to be taken as genuine uncertainties in the prediction for the Higgs cross section. For such integrated quantities we expect the standard DGLAP approach implemented in KMR to give a reasonably predictive answer, and we conclude that the CCFM and LDC based densities parameterizations simply are not well enough constrained to give reasonable predictions for Higgs production at the LHC. The problem is that the Jung and LDC densities have only been fitted to  $F_2$  at HERA which means mainly small  $x$  and  $Q^2$ , while for Higgs production we have much larger scales and through evolution we are also sensitive to the large- $x$  behavior at lower scales, which is not well constrained.

If the LDC and Jung densities are not constrained enough to predict inclusive Higgs production at the LHC, it is unlikely that they are able to say anything predictive about exclusive Higgs production. However, although the normalization is uncertain, it may still be possible that these densities have some predictive power on the  $k_{\perp}$ -dependence of the uPDF. In figure 5 we show the normalized  $k_{\perp}$ -distribution of a centrally produced Higgs at the LHC as predicted by the different uPDFs, and we see that the differences are large, but not unreasonable. We find that the spectra are harder for *leading* and Jung-1 than for *standard*, *gluonic* and Jung-2, which is expected since the former only have singular terms in the gluon splitting function which allows the gluon to radiate more.

## 4.2 Exclusive Higgs production

Although we do not believe that the LDC/Jung uPDFs can be used to give any prediction



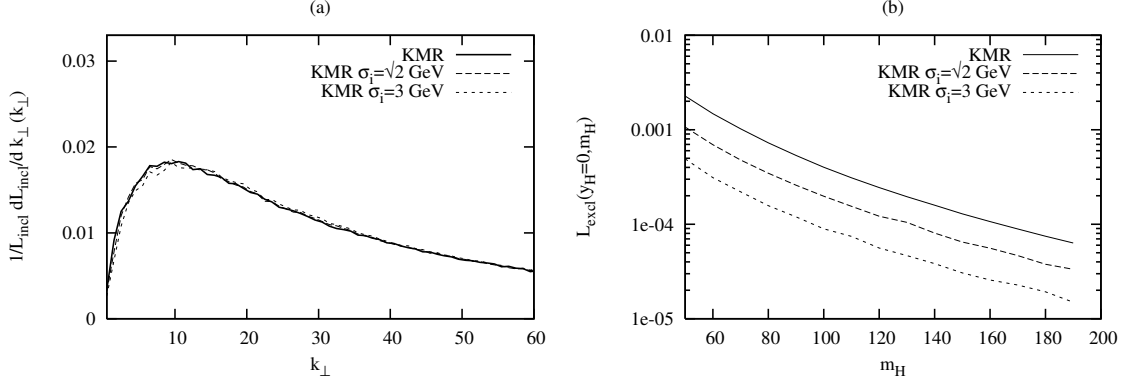
**Figure 6:** The ratio of the exclusive to inclusive luminosity function at fixed central rapidity as a function of  $m_H$ . The lines are the same as in figure 3.

for neither the inclusive or exclusive luminosity, it is not unlikely that they actually have some predictive power on the ratio of the two. We saw above that the normalized  $k_\perp$ -distribution of the Higgs looks reasonable. In addition, although the uPDFs enters to the power 4 in the exclusive luminosity function, according to eqs. (3.7) and (3.8), the high scale uPDF only enters with power 2 while the other two powers depend on lower scales where the uPDFs may be better constrained. Hence, the uncertainty from the evolution to high scales may cancel in the ratio.

In figure 6 we show the ratio between the exclusive and the inclusive luminosity functions for fixed central rapidity as a function of  $m_H$  according to eqs. (2.1) and (4.2). There are clearly large differences, probably too large to be attributed to anything else than that the LDC and Jung densities simply are not constrained enough to give any reasonable predictions.

We know that the inclusive luminosity in eq. (2.1) is mostly sensitive to  $k_\perp$ -values around a couple of GeV, and we can see that the Jung-1 is much lower than Jung-2 which can be attributed to the fact that Jung-1 has a harder  $k_\perp$ -distribution than Jung-2 reducing the density in this region relative to higher  $k_\perp$ . Similarly *leading* is much lower than *standard* and *gluonic* and again the former has a harder  $k_\perp$ -distribution than the two latter. But since there are large differences in general between LDC and Jung we cannot say that the differences simply does not come from the fact that all these uPDFs are too unconstrained.

To focus on the uncertainties in the  $k_\perp$  distribution of the uPDFs, we instead concentrate only on the KMR densities, where we know that the overall normalization is well constrained, and study what happens if we simply shift the  $k_\perp$  distribution slightly, while keeping the integrated PDF fixed. We know that the  $k_\perp$ -spectrum of the Z and W at the Tevatron can be well described by standard DGLAP based parton showers if an Gaussian



**Figure 7:** (a) The normalized  $k_{\perp}$ -distribution of a central Higgs produced at LHC as predicted by using KMR uPDFs with different Gaussian intrinsic  $k_{\perp}$  added. (b) The exclusive luminosity function (eq. (2.1)) at fixed central rapidity as a function of  $m_H$  calculated using KMR uPDFs with different Gaussian intrinsic  $k_{\perp}$  added. In both cases the full line corresponds to no intrinsic  $k_{\perp}$  and dashed and dotted lines corresponds to a Gaussian intrinsic  $k_{\perp}$  with a width of  $\sqrt{2}$  and 3 GeV respectively.

intrinsic  $k_{\perp}$  with a width of a couple of GeV is added to the incoming quarks. Judging from figure 1 it does not seem unlikely that the shape would be better reproduced if the KMR uPDF was modified in the same way.

In figure 7a we see the effect of such an intrinsic  $k_{\perp}$  on the  $k_{\perp}$ -distribution of a 120 GeV Higgs at fixed central rapidity at LHC. We here use a larger intrinsic  $k_{\perp}$  than would be needed at the Tevatron which, as discussed above, is not unreasonable since we are here dealing with gluons rather than quarks and we have much smaller  $x$ -values, allowing for more unordered evolution. Still the effect on the Higgs spectrum is rather moderate, especially compared to the effects in figure 5. However, the effect on the exclusive luminosity is large, as can be seen in figure 7b. Adding a Gaussian intrinsic  $k_{\perp}$  with a width of 3 GeV reduces the luminosity by approximately a factor 5. And we conclude that the exclusive production of Higgs at the LHC is very sensitive to the small- $k_{\perp}$  distribution of the unintegrated gluon.

## 5. Conclusions

The main conclusion of this article is a negative one. The predictive powers of the unintegrated gluon density functions as fitted only to small- $x$  HERA data is very poor when applied to exclusive Higgs production at LHC. In fact, not even inclusive Higgs production at the LHC is well constrained with these uPDFs. However, looking at the qualitative differences between these uPDFs we can learn something about where the uncertainties come from. Here we have argued that there are problems not only with the overall normalization of the uPDFs at the high scales under consideration, but also the actual  $k_{\perp}$ -distribution at small  $k_{\perp}$  is important. The reason is clearly visible in the  $k_{\perp}$ -integration in the exclusive luminosity function, where the main contribution comes from transverse momenta in the region of a couple of GeV.

The situation is quite different when it comes to the uPDF derived from the integrated gluon density using the KMR prescription. Here we believe the overall normalization to be well determined by the global PDF fits, and the predictions for inclusive Higgs production should be trustworthy. However, the prediction for the distribution of small  $k_\perp$  values is less certain and there is evidence that eg. the  $k_\perp$  distribution for W and Z production at the Tevatron obtained from the KMR prescription is a bit too high for small  $k_\perp$ . This is consistent with the behavior of DGLAP-based parton shower approaches, which are closely related to the KMR approach, which typically need an additional gaussian intrinsic  $k_\perp$  of one or two GeV to reproduce W and Z transverse momentum spectra. We have found that introducing an intrinsic  $k_\perp$  in the KMR uPDF in the calculation of the exclusive luminosity function will give a clear reduction.

We will not try to use our findings to make an estimate of the uncertainties involved in the KhMR predictions for the exclusive Higgs production at the LHC and elsewhere. Clearly there is a need to find better experimental observables to constrain the (off-diagonal) unintegrated gluon density before we can make precise predictions. We do feel that the published KhMR predictions may be too high, but clearly they should give the right order of magnitude, and the prospect of using the exclusive process to study the Higgs at the LHC is still a very interesting one.

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